This will be recognized as being equal to the lateral stiffness of a beam without axial load, since the tip deflection y of such a beam under lateral load F at the tip is given by

$$y_t = FL^3/3EI \tag{6}$$

Using the beam-column stiffness $K(P)$ in place of K in the Timoshenko and Gere formula, Eq. (1) gives

$$P = \frac{P(CL)}{L} \frac{1}{(\tan\beta L/\beta L - 1)} \tag{7}$$

or

$$(\tan\beta L/\beta L) - 1 = C \tag{8}$$

which is exactly Parnes' Eq. (4). For small values of C, we have the buckling of the rigid link in rotation as the predominating failure mode. From Eqs. (1) and (5) of this Comment or by use of the series expansion Eq. (4) in Eq. (8), this load is

$$P \approx (3EI/L^3)CL \tag{9}$$

This illustrates the "paradoxical" behavior at small values of C noted by Parnes and is seen here to be essentially the same physical phenomenon dealt with by Timoshenko and Gere.

As C approaches infinity, buckling of the elastic column predominates, and from Eq. (8) the critical load is given by $\beta L = \pi/2$, which is the classical buckling load of the cantilever column. As Parnes showed, consideration of the finite stiffness of the link BC introduces no interaction with the phenomena just discussed but merely requires the additional consideration of a possible buckling condition of the link BC, considered to be simply supported at both ends.

References

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Reply by Author to A. H. Flax

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THE author wishes to thank Dr. Flax for his pertinent discussion of the apparent "paradox" and for bringing to his attention an explanation of the paradox by considering the lateral deflection of a rigid link resisted by a spring of effective stiffness $K(P)$. While this explanation is quite correct and ingenious, the author believes the explanation given in the previous discussion[1] to be more direct, as it derives immediately from considerations of statics.

Although it is well known that systems of rigid links connected in various ways by means of elastic springs often lead to increasing values of critical loads with member length, the author believes that this phenomenon had not been previous shown to exist for cases of elastic columns that undergo flexure.

References

[1] Parnes, R., "Reply by Author to D.J. Johns," *AIAA Journal*, Vol. 16, Sept. 1978, p. 1115.

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Comment on "Effects of Radial Appendage Flexibility on Shaft Whirl Stability"

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WILGEN and Schlack[1] investigate the stability of a rotating shaft with two opposite radial booms attached to it (see Fig. 1). They give the stability limits as a function of the flexibilities of the components and of other system parameters. In the following it is shown that 1) analytical solutions can be given for the shaft deflections which are approximated by a series in Ref. 1, 2) simple and accurate numerical solutions do exist for the deflections of the radial booms, which are approximated by one single "comparison function" in Ref. 1, and 3) the out-of-plane stability of the system is strongly dependent on the flexibility of the radial booms and not independent of that parameter as stated in Ref. 1. The in-plane stability limits given in Ref. 1 will be recalculated on the basis of the preceding comments and new stability limits will be given for the out-of-plane motion.

The existence of analytical solutions for rotating shafts is well known.[2] Less known perhaps is the fact that analytical solutions for rotating symmetric shafts can be derived from the well-known solutions for nonrotating shafts (i.e., booms) by pure coordinate transformation.[3] This is a direct consequence of the fact that a rotation with respect to the shaft axis is not reflected in the differential equations for a symmetric shaft if the system of reference is not affected by this rotation. In general, however (but not always), rotating coordinates are used with rotating shafts. That is why the spin appears in the equations. The fact that the differential equations (and consequently the dynamic behavior) of a rotating symmetric shaft are not affected by the speed of rotation is only true for conservative systems. Inner damping forces, for instance, do depend on the shaft's rotation, and it is only due to these forces that a shaft becomes unstable at a certain critical speed.

For radial booms, on the other hand, analytical solutions are not known because the underlying differential equations have nonconstant parameters arising from the nonconstant axial (centrifugal) force. However, numerical solutions of the differential equations can be given with a precision that depends only on computer accuracy and not on modal truncation or discretization effects. Here the boom deflections are expressed by a power series.

The recurrence law for the generation of the coefficients of the series follows directly from the coefficients of the dif-

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